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A COMPARISON BETWEEN THE OBSERVED AND PREDICTED AMPLITUDE OF THE 24^{TH} SOLAR CYCLE

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Abstract. In present work we compared the measured and predicted amplitudes of the 24^{th} solar cycle. The modified minimum–maximum method, belonging to the precursor class of methods, was applied to the smoothed monthly sunspot number values (the "old" data set, used before the change introduced on July 1^{st}, 2015). The maximum of the 24^{th} solar cycle occurred in April 2014 with an amplitude of $R = 82$ and this observed value is very close to our mean predicted value $R = 83$. The maximum was significantly weaker than in several previous cycles. Additionally, a curious solar activity minimum of 2008, between the solar cycles no. 23 and no. 24 was analysed, as well as the shape of the maximum profile. The maximum of the 24^{th} solar cycle had a double-peak, the second one being higher than the first one. The obtained results represent a strong indication that the minimum–maximum method is a reliable tool for the solar cycle prediction, using data available already 3 years before the preceding minimum of solar activity.

Key words: Solar cycle prediction

1. Introduction

Solar variability can be divided into short-term and long-term variations. The short-term variations include scales of less than 10 years, *i.e.* of about the length of one solar activity cycle. Their influence is mostly described in terms of the space weather and space climate conditions in the solar system. The long-term variations are effective on time scales longer than 10 years and might have important influence on the Earth's climate. The forecast of solar activity on the scale of years and decades is very important for planing future space missions, both manned and robotic, to the Moon and Mars. Further, note that both the short-term and long-term variations control the physical state of the Earth's magnetosphere, influencing the geomagnetic field and is the best reflected in the geomagnetic observatory data (Verbanac *et al.*, 2015).

In addition to this practical, applied aspect, the reliability analysis of solar cycle prediction methods can help to better understand characteristics of the solar activity cycle. Thus, it could improve distinguishing between the stochastic (random) and non-linear (chaotic) effects besides the well-known periodic (cyclic) pattern (Wilson, 1994; Letellier *et al.*, 2006; Gilmore and Letellier, 2007; Aguirre *et al.*, 2008).

In the present paper we compare the observed and predicted characteristics of the 24$^{\text{th}}$ solar cycle. We focus on the amplitude of the maximum of the 24$^{\text{th}}$ solar cycle.

The theoretical attempts for solar cycle prediction are based on various treatments of the dynamo process. These studies can be divided into two subgroups. Those investigations which analyze the earlier temporal variations of the cycles and extrapolate the observed regularities or trends can be regarded as extrapolation methods. They should tacitly assume that an unspecified dynamo regime is persistent, therefore the successes or failures of forecasts are indications for additional processes besides the dynamo mechanism. The other group of studies follows the precursor method. These are based on some physical considerations about the succession of events, and the precursors of highest predictive power may become important ingredients of the dynamo models.

In our previous paper (Brajša *et al.*, 2009) a general overview of the relevant literature is given. In the meantime a rather huge number of papers about the current topic was published. Here we only emphasize recent developments which were summarized by Hathaway (2009), Petrovay (2010) and Pesnell (2012). We make now a short note on terminology and notation. In this paper we use the International sunspot number (also called the relative sunspot number or the Wolf number), denoted by R or ISSN. We use the monthly smoothed values in present analysis. We note that on July 1$^{\text{st}}$ 2015 a major change of the sunspot number data set was performed including change of the relative sunspot number[1]. In present analysis we use the "old" sunspot number data set, which is also available at the SIDC web page in the Archive section.

In this work we use one of the precursor methods, namely the modified minimum–maximum method. The paper is organized as follows: in Section 2. we give a brief overview of the basic characteristics of the 24^{th} solar cycle, in Section 3. we describe the modified minimum–maximum method, present its results, and finally in Section 4. we compare the real, observed behavior with the predicted one.

2. Basic Characteristics of the 24^{th} Solar Cycle

Now, when the maximum of the 24^{th} solar cycle has passed, we can summarize its main characteristics. The maximum was a double-peaked one (as discussed by Kilcik and Ozguc, 2014), the first peak having $R_1 = 66.9$ in February 2012 and the second peak being $R_2 = 81.9$ in April 2014. We note that these quantities are the monthly smoothed relative sunspot number values, the same parameter which we use throughout the whole paper. Thus, we assume that the actual maximum had the amplitude $R = 81.9 \approx 82$. We will compare this measured result with the predicted values in next two Sections of this paper.

In Figs. 1 and 2 we can follow the development of the 24^{th} solar cycle[2]. The smoothed monthly sunspot numbers are plotted as a function of time in months, relative to the minimum epoch which occurred in the last two calendar months of the year 2008: $T_{\text{min}} = 2008.9$; $R_{\text{min}} = 1.7$. In Fig. 1 (Fig. 2) the curve representing the actual solar cycle is compared with similar curves for the previous 10 (20) solar cycles. From Fig. 1 we can see that the solar activity minimum in 2008 was indeed broader and deeper than most of the last 10 solar cycles, making the minimum between 23^{rd} (the maximum occurred in April 2000) and 24^{th} solar cycles relatively curious. It is however not completely unusual, which can be seen in Fig. 2. The cycles with minima below the current value, even with 0.0 in some cases, took place in the beginning of the 19^{th} century. Therefore, we can conclude that the solar cycle no. 24 has some similarities with the cycles at the beginning

[1] `http://sidc.oma.be/silso/`
[2] `http://www3.kis.uni-freiburg.de/~hw`

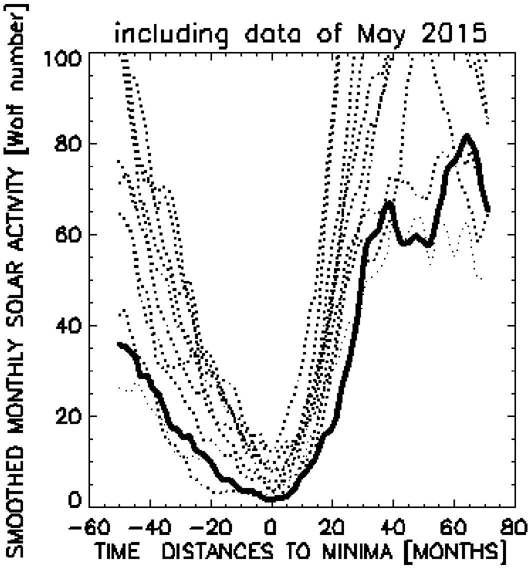

Figure 1: Monthly smoothed relative sunspot number for current (bold) and 10 previous cycles (dotted lines). The last measured value for November 2014, which was available after May 2015, is included.

of both the 19th and the 20th century in agreement with the Gleissberg cycle with the period 90 – 100 years.

The period of the solar cycle, measured as the time span between the maximum of the 23rd and the 24th solar cycles, is relatively long amounting exactly to 14.0 years. We can summarize, that the minimum was rather low and broad, the maximum modest and double-peaked (with the second peak higher than the first one) and that the length of the cycle is relatively long. We note that some curious characteristics of the last solar minimum were also reported by Tapping and Valdés (2011) who used 10.7 cm solar radio flux data.

3. The Minimum–Maximum Method

The minimum–maximum method (*e.g.* Wilson, 1990a) is based on a linear relationship between the smoothed monthly relative sunspot number in the minimum ($R_{\rm min}$) and in the maximum epoch ($R_{\rm max}$) of solar cycles for which data exist (SIDC-team, 1749–2015). In the present analysis we apply

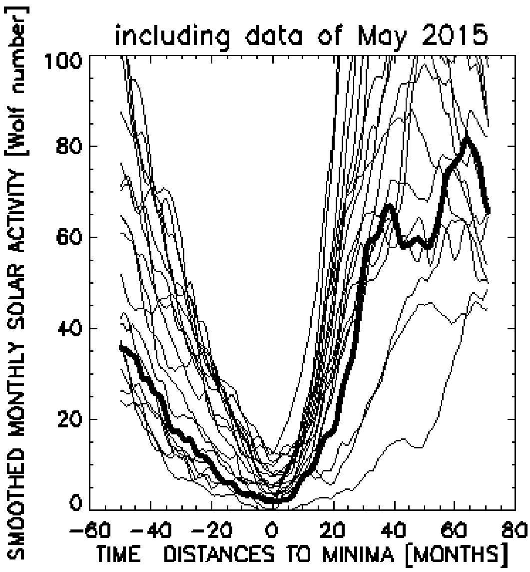

Figure 2: The same as in Fig. 1, but including data for the last 20 solar cycles.

a modified version of this method using not only the minimum activity year, but years before and after the solar activity minimum. The correlation coefficient, CC, is investigated as a function of the time offset in years. From Fig. 3 it can be seen that there is a non-trivial maximum in the CC 3 years before the minimum (after the minimum the CC sharply rises, as the cycle moves toward the maximum). We obtain the following result for all solar cycles 1–23 (Fig. 3):

$$R_{\max} = (45 \pm 12) + (1.6 \pm 0.3) \cdot R_{\min-3}, \qquad CC = 0.81 \qquad (1)$$
$$R_{\max} = (78 \pm 14) + (5.9 \pm 1.9) \cdot R_{\min}, \qquad CC = 0.56 \qquad (2)$$
$$R_{\max} = (62 \pm 16) + (2.8 \pm 0.8) \cdot R_{\min+1}, \qquad CC = 0.62 \qquad (3)$$
$$R_{\max} = (49 \pm 10) + (1.1 \pm 0.1) \cdot R_{\min+2}, \qquad CC = 0.86 \qquad (4)$$
$$R_{\max} = (39 \pm 7) + (0.8 \pm 0.1) \cdot R_{\min+3}, \qquad CC = 0.93 \qquad (5)$$
$$R_{\max} = (22 \pm 6) + (0.9 \pm 0.1) \cdot R_{\min+4}, \qquad CC = 0.96 \qquad (6)$$

Now we consider the time of the last solar minimum that occurred in November-December 2008, when the smoothed monthly relative sunspot number had the same, lowest value of 1.7 in both months. Also, the cor-

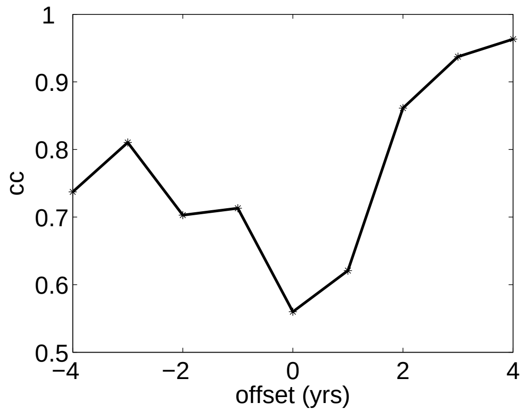

Figure 3: The correlation coefficient of the minimum–maximum relationship with the time offset in years. Solar cycles 1–23 were analyzed using smoothed monthly values of the relative sunspot number.

responding mean value for the two months 3 years before (in 2005) and 4 successive years thereafter (2009-2012) were found. For those six cases, we obtain the maximum of the 24$^{\rm th}$ solar cycle $R_{\rm max}^{\rm (min-3)} = 84 \pm 14$, $R_{\rm max}^{\rm (min)} = 88 \pm 14$, $R_{\rm max}^{\rm (min+1)} = 84 \pm 17$, $R_{\rm max}^{\rm (min+2)} = 80 \pm 10$, $R_{\rm max}^{\rm (min+3)} = 90 \pm 8$ and $R_{\rm max}^{\rm (min+4)} = 76 \pm 7$.

Finally, we repeat this procedure excluding the solar cycle no. 19, since it is well known that it was an unusual case, maybe even a real outlier (Wilson, 1990b; Temmer *et al.*, 2006):

$$R_{\rm max} = (49 \pm 12) + (1.5 \pm 0.3) \cdot R_{\rm min-3}, \qquad CC = 0.79 \qquad (7)$$
$$R_{\rm max} = (67 \pm 11) + (6.9 \pm 1.5) \cdot R_{\rm min}, \qquad CC = 0.73 \qquad (8)$$
$$R_{\rm max} = (62 \pm 14) + (2.6 \pm 0.7) \cdot R_{\rm min+1}, \qquad CC = 0.65 \qquad (9)$$
$$R_{\rm max} = (52 \pm 10) + (1.0 \pm 0.2) \cdot R_{\rm min+2}, \qquad CC = 0.83 \qquad (10)$$
$$R_{\rm max} = (41 \pm 7) + (0.8 \pm 0.1) \cdot R_{\rm min+3}, \qquad CC = 0.92 \qquad (11)$$
$$R_{\rm max} = (22 \pm 7) + (0.9 \pm 0.1) \cdot R_{\rm min+4}, \qquad CC = 0.95 \qquad (12)$$

Similar as before, for the six cases, we obtain for the maximum of the 24$^{\rm th}$ solar cycle $R_{\rm max}^{\rm (min-3)} = 85 \pm 13$, $R_{\rm max}^{\rm (min)} = 79 \pm 11$, $R_{\rm max}^{\rm (min+1)} = 83 \pm 15$, $R_{\rm max}^{\rm (min+2)} = 81 \pm 8$, $R_{\rm max}^{\rm (min+3)} = 90 \pm 8$ and $R_{\rm max}^{\rm (min+4)} = 76 \pm 8$.

In summary, the minimum–maximum method gives a mean value of

$R_{\max} = 83 \pm 7$ for the peak relative sunspot number for the 24^{th} solar cycle. Finally, we would like to emphasize the result obtained for the correlation three years after the minimum. According to Hathaway *et al.* (1994) this should be a fairly reliable forecast. For the present cycle this gives the largest predicted value $R_{\max} = 90 \pm 8$.

4. Discussion and Conclusions

In Sect. 2. we presented results of the prediction of modified minimum–maximum method. The mean value for all subcases, $R_{\max} = 83 \pm 7$ is very close and is consistent with the actual measured value $R_{\max} = 82$. Thus, the minimum–maximum method is a rather useful prediction tool for the forecast of the solar cycle activity maximum.

Now we make two comments on our results. First, the calculation using data already 3 years before the minimum gives rather accurate prediction of the subsequent maximum. This is in agreement with the largest correlation coefficient of the minimum–maximum relationship (Fig. 3). Second, the forecast based on data 3 years after the minimum gives the result which has the largest difference to the measured value, at least for the 24^{th} solar cycle. This is in disagreement with the claim of Hathaway *et al.* (1994).

For predicting the amplitude of the 24^{th} solar cycle maximum various subtypes of the precursor method were used (Svalgaard *et al.*, 2005; Dabas *et al.*, 2008; Obridko and Shelting, 2008; Ahluwalia and Ygbuhay, 2009; Bhatt *et al.*, 2009; Brajša *et al.*, 2009; Tlatov, 2009a,b; Wang and Sheeley, 2009; Yoshida and Yamagishi, 2010; Kakad, 2011; Podladchikova and van der Linden, 2011; Uzal *et al.*, 2012). However, it should be noted that most of those works overestimated the amplitude of the 24^{th} solar cycle maximum. On the other hand, Svalgaard *et al.* (2005), Yoshida and Yamagishi (2010), Kakad (2011), and Uzal *et al.* (2012) predicted approximately correct the modest amplitude of the last solar maximum, while Podladchikova and van der Linden (2011) underestimated the measured maximum amplitude. We emphasize that the prediction of Svalgaard *et al.* (2005) was based on the correlation of the strength of the Sun's axial magnetic dipole moment 3 years before the activity minimum with the amplitude of the subsequent solar maximum. It is interesting and probably not accidental that our method (although different from their) yields the highest correlation coefficient and the very good prediction (in agreement with the measured amplitude) when also the data from 3 years before the minimum were used.

We can conclude that the minimum–maximum method, here presented in a modified form, represents a rather reliable forecast tool for predicting the amplitude of the subsequent solar maximum. The method makes the prediction fairly well using data even 3 years before the preceding minimum. Hence, in the moment when the minimum epoch is estimated well, or just after the minimum has passed, the fairly reliable prediction of the next maximum is possible.

Finally, it is still not fully clear what are the chaotic properties of solar activity. In fact, the non-linear effects, mostly neglected in the forecast procedures, would have very important implications for predictions. There are many possibilities where these non-linear effects could come from, *e.g.* in considering the chaotic solar dynamo where non-linear differential equations with higher order terms of specific physical quantities appear (Ossendrijver, 2003; Rüdiger and Hollerbach, 2004; Wilson, 1994; Gilmore and Letellier, 2007; Aguirre *et al.*, 2008; Letellier *et al.*, 2006). The method presented in this work does not take into account the non-linear effects; this should be left for other methods.

In further work we plan to address some of the open questions, especially repeating the whole analysis for the northern and the southern hemisphere separately and using the alternative time series of sunspot number data. We also plan to use some other methods like the Autoregressive moving average model (ARMA), Damped random walk (DRW) model and apply the Hurst analysis to the available data.

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