Ship roll control and energy harvesting using a U-tube anti-roll tank

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Abstract
In the traditional design of passive anti-roll tanks (ART), the energy associated with the tank fluid motion is dissipated. The work reported in this paper explores the possibility that this energy is instead harvested. It is analytically determined in the paper how the natural frequency and the damping ratio of a U-tube ART should be tuned to maximise the power absorbed by the tank fluid. To this end, a perfectly flat spectrum of the moment representing the excitation of the ship by waves has been assumed. It is found that a tuning that maximises the power absorbed by the ART fluid motion also minimises the average kinetic energy of the ship roll. This is a result of the fact that the power input of the wave excitation moment into the ship plus tank system does not depend on how the ART is tuned. Therefore, different tunings of the ART natural frequency and damping ratio only affect the distribution of the power dissipated by the ship roll damping and the power absorbed by the ART.

1 Introduction

Whereas all angular degrees of freedom (roll, pitch and yaw) are important for sea-keeping characteristics of any ship, roll motion is known to be the critical one [1–3]. This is because ship roll is typically lightly damped and the restoring moment of the ship is small in the cross-plane in comparison with the other planes. As a result, excessive roll can occur under unfavourable or extreme sea conditions. This can lead to reduced effectiveness of the crew, damaged or lost cargo, limited operability of the on-board equipment, or even to catastrophic sea accidents which include capsizing of the ship and the loss of human lives.

Roll reduction devices include keels, fin stabilizers, [4,5], rudders, [6], gyro stabilizers [7,8], azimuthing propellers, [9], and anti-roll tanks (ARTs), [10–14]. Among these devices, anti-roll tanks have raised a considerable attention. Contrary to fin stabilisers, ARTs are effective at low forward speeds. This is relevant, for example, for offshore service vessels such as the wind farm installation vessels, [15]. ARTs do not cause highly concentrated loads, like for example gyro stabilisers, and do not require complicated mechanisms such as Weis–Fogh flapping fin stabilizers, [16]. The operational costs related to ARTs are low. A drawback of ART technology is a remarkable space required for the installation which reduces the space available for transport of cargo. Also, the tank free surface effect reduces the metacentric height of the ship.

Probably the first attempt to use a fluid-filled tank to control the ship roll was at the end of the 19th century [11,17,18]. In 1911 an ART in the shape of a U-tube was proposed, having the horizontal channel below the centre of gravity of the ship, [10]. U-tube ARTs were installed in over 1,000,000 tons of German shipping before WWII, [12]. By 1975 nearly two thousand ships of various types had been fitted with different types of tank stabilisation systems, [19]. Later on, many authors developed mathematical models for analysis of seakeeping of ships equipped with ARTs, and evaluated the ART performance with respect to the control of roll motion, either actively or passively, [20–24], including the control of parametric roll [13,14,25]. For example, a linearized two-degree-of-freedom (dof) model for the analysis of the roll of a ship equipped by a U-tube ART has been developed by Stigter, [26]. The two dofs are the angle of the ship roll and the angle of the free surface of the tank fluid. The author also discussed the validity of the model by a comparison with experimental results on a scaled model.

Contemporary anti-roll tanks can be designed either in the form of a free-surface ("flume") tank or in the form of a U-tube tank, [5]. A theoretical and experimental comparison of the two designs can be found in [19,27]. With free-surface tanks, an existing liquid reservoir on board can be adapted to perform additional function, that is, to impede the roll motion [19,28]. This may be done without major interventions into the tank construction, apart from installing flooded baffles in order to the increase the damping of the oscillatory fluid sloshing [29,30]. The control of the period of sloshing requires adjusting the level of the liquid in the tank: the higher the level, the lower the period, [19]. On the other hand, U-tube tanks are designed so as to have separated port and starboard portions of the tank connected by a channel between them, [10]. The damping ratio is normally tuned through the size and shape of an orifice (a valve) in the connecting channel of the U-tube. The natural frequency is controlled by a careful sizing of the device. This is predominantly done through choosing the width of the two portions of the tank and their separation distance, which enable adjusting the moment of inertia of the free surface with respect to the symmetry line of the tank installation. Fine tuning can be achieved with the level of filling of the U-tube and dimensioning the height of the connecting channel. If properly designed, a U-tube tank can reduce the roll motion of a ship excited by waves in a way similar to how Tuned Vibration Absorbers (TVA) reduce the response of a mechanical structure to simple harmonic or stochastic forcing, [31–34]. In fact, apart from proposing his U-tube anti roll tank, Frahm had also patented in 1911 an entirely mechanical “device for damping vibrations of bodies”, [31]. This device, effectively a TVA, can be modelled as a mass suspended to the controlled structure through a spring and a damper. It creates an anti-resonance condition at its natural frequency. A U-tube ART works in a similar manner and it can be represented by a simplified mechanical scheme with lumped parameters and linear motions. The equivalent mechanical scheme, however, is somewhat more complex than the TVA scheme [35]. Thus the various optimum tuning strategies developed for TVAs [36–40], cannot be directly used to tune ARTs.

The natural frequency and the damping ratio of ARTs have been typically tuned according to the H_∞ criterion. This criterion aims at minimising the amplitude of the ship roll angle at wave excitation frequencies where maxima of the roll response occur. The roll of a ship without the ART installation is characterised by one such frequency, roughly corresponding to the roll natural frequency. However, the roll of a ship equipped with a lightly damped ART is characterised by two resonance frequencies, due to the additional degree of freedom of the liquid in the tank. Then the optimum natural frequency and damping ratio of the tank can be determined using the H_∞ criterion by employing the so-called “fixed point theory”, originally introduced by Den Hartog [32] for the control of mechanical vibrations, and adapted to the problem of tuning a U-tube ART for ship roll control by Stigter [26]. Using this criterion minimises (and equalises) the roll response at the two resonance frequencies.

The energy associated with the tank fluid motion in traditional U-tube ARTs is normally dissipated, mostly through the viscous damping at the valve in the channel connecting the port and starboard reservoir of the U-tube. The investigation reported in this paper explores the possibility that this energy is instead harvested.

In fact, the specific aim of the work presented in this paper is twofold. On one hand, it is investigated how the natural frequency and damping ratio of an ART should be tuned to maximise the power absorbed by the ART. On the other hand, it is investigated how this tuning corresponds to a tuning that minimises the kinetic energy of the ship roll. The two tuning criteria are compared assuming a perfectly flat spectral distribution of the moment representing the excitation of the ship by waves. Although this assumption is far from the real conditions at sea, it facilitates analytical estimation of the optimum natural frequency and damping ratio of the ART.

The paper is structured as follows. In Section 2 the mathematical model for the analysis of the roll of a ship equipped with an ART is presented. Equations of motion are given and the balance of the power input into the ship plus tank system with the power dissipated by the system is introduced. In Section 3 the roll of a ship equipped with an ART is discussed assuming the flat spectrum of the roll excitation moment. Analytical expressions for the natural frequency and damping ratio of the ART either to minimise the average kinetic energy of the ship roll, or to maximise the power absorbed by the ART installation are derived.

2 Mathematical model

2.1 Equations of motion

The schematic representation of the ship equipped with an anti-roll tank is shown in Figure 1.

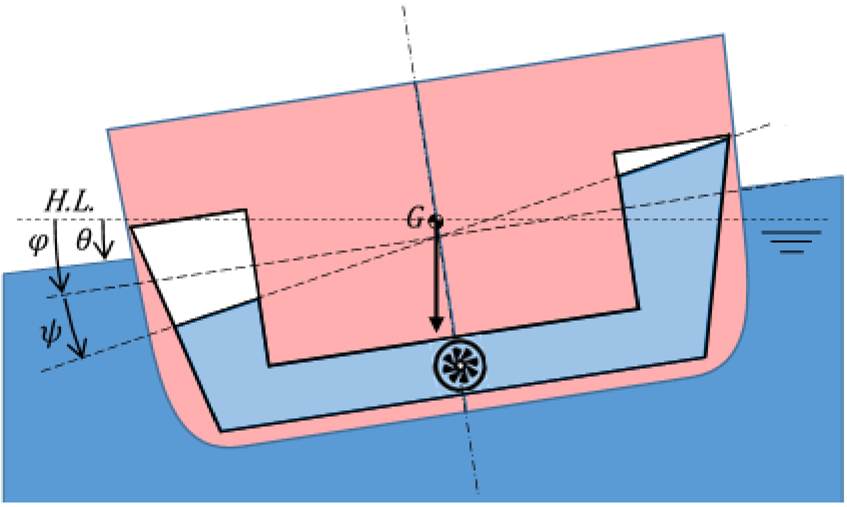

Figure 1: The scheme of a ship equipped with an anti-roll tank

According to Stigter's linearized model of a ship plus tank, [26], equations of motion can be written as :

$$a_1\ddot{\varphi} + a_2\dot{\varphi} + a_3\varphi + c_1\ddot{\psi} + c_3\psi = m_\theta \,, \qquad (1)$$

$$c_1\ddot{\varphi} + c_3\varphi + b_1\ddot{\psi} + b_2\dot{\psi} + b_3\psi = 0 \,, \qquad (2)$$

where φ is the ship roll angle, ψ is the tank fluid angle measured relative to the ship (Figure 1), an m_θ is the wave excitation moment.The coefficients $a_i, b_i,$ and c_i are:

$a_1 =$ the moment of inertia of the ship+ART system, assuming a "frozen" tank fluid (kgm^2),

$a_2 =$ the linear roll damping coefficient of the hull (Nms/rad),

$a_3 =$ the righting moment of the ship+ART system assuming a "frozen" tank fluid (Nm/rad),

$b_1 =$ the moment of inertia of the tank fluid assuming a motionless ship (kgm^2)

$b_2 =$ the total linear angular damping coefficient between the tank fluid and the tank walls (Nms/rad)

$b_3 =$ the restoring moment of the tank fluid assuming a motionless ship (Nm/rad)

$c_1 =$ the moment of inertia of the "frozen" tank fluid (kgm^2)

$c_3 = b_3$, [26].

All moments of inertia, restoring moments and the righting moment are with reference to the centre of gravity of the ship plus ART system ([26]). Full details on how to calculate all model coefficients for a given ship can be found in [26]. A discussion on the assumptions required to derive the linearized model and its experimental validation can also be found in the reference.

The equations of motion (1) and (2) can be written in the matrix form as:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}\,, \tag{3}$$

where **M** is the generalised inertia matrix, **K** is the generalised restoring moment matrix, **C** is the damping matrix, $\mathbf{x}(t)$, $\dot{\mathbf{x}}(t)$, $\ddot{\mathbf{x}}(t)$ are the angular displacement, velocity and acceleration column vectors respectively, and $\mathbf{F}(t)$ is the generalised excitation moment column vector. These matrices/vectors are given by the following expressions:

$$\mathbf{M} = \begin{bmatrix} a_1 & c_1 \\ c_1 & b_1 \end{bmatrix},\ \mathbf{C} = \begin{bmatrix} a_2 & 0 \\ 0 & b_2 \end{bmatrix},\ \mathbf{K} = \begin{bmatrix} a_3 & c_3 \\ c_3 & b_3 \end{bmatrix}, \tag{4 a-c}$$

$$\mathbf{x} = \begin{bmatrix} \varphi(t) \\ \psi(t) \end{bmatrix}, \mathbf{F} = \begin{bmatrix} m_\theta(t) \\ 0 \end{bmatrix}. \tag{5 a-b}$$

Assuming a simple harmonic oscillation of the wave excitation moment, $m_\theta = \Re\left\{\hat{M}_\theta\, \mathrm{e}^{-\mathrm{i}\omega t}\right\}$, the steady state forced response of the ship and the tank are also simple harmonic, $\varphi = \Re\left\{\hat{\Phi}\, \mathrm{e}^{-\mathrm{i}\omega t}\right\}$, and $\psi = \Re\left\{\hat{\Psi}\, \mathrm{e}^{-\mathrm{i}\omega t}\right\}$, where $\mathrm{i} = \sqrt{-1}$ and $\Re$ stands for the real part. Therefore Eq. (3) can be written as:

$$\mathbf{S}(\mathrm{i}\omega)\mathbf{x}(\mathrm{i}\omega) = \mathbf{F}(\mathrm{i}\omega), \tag{6}$$

where $\mathbf{S}(\mathrm{i}\omega)$ is a dynamic stiffness matrix with the following form:

$$\mathbf{S}(\mathrm{i}\omega) = -\omega^2\mathbf{M} + \mathrm{i}\omega\mathbf{C} + \mathbf{K}\,. \tag{7}$$

The solution of Eq. (6) can be obtained by inversion of the dynamic stiffness matrix $\mathbf{S}(\mathrm{i}\omega)$ as

$$\mathbf{x}(\mathrm{i}\omega) = \mathbf{S}^{-1}(\mathrm{i}\omega)\mathbf{F}(\mathrm{i}\omega)\,. \tag{8}$$

Differentiating Eq. (8) in order to obtain ship roll and tank fluid angular velocities results in the following expression:

$$\dot{\mathbf{x}}(\mathrm{i}\omega) = \mathbf{Y}(\mathrm{i}\omega)\mathbf{F}(\mathrm{i}\omega), \tag{9}$$

where $\dot{\mathbf{x}}(\mathrm{i}\omega) = \mathrm{i}\omega\,\mathbf{x}(\mathrm{i}\omega)$ is a vector containing ship and tank free surface angular velocities and $\mathbf{Y}(\mathrm{i}\omega) = \mathrm{i}\omega\,\mathbf{S}^{-1}(\mathrm{i}\omega)$ is the angular mobility matrix (2×2) containing frequency response functions (FRFs) between angular velocities and excitation moments:

$$\mathbf{Y}(\mathrm{i}\,\omega) = \begin{bmatrix} Y_{\dot{\varphi},m_\theta} & Y_{\dot{\varphi},m_\psi} \\ Y_{\dot{\psi},m_\theta} & Y_{\dot{\psi},m_\psi} \end{bmatrix}. \tag{10}$$

Taking into account Eq. (5b), the steady-state angular velocity responses of the ship and tank system to the moment exciting the ship, are the two FRFs located in the first column of the angular mobility matrix $Y_{\dot{\varphi},m_\theta}$, and $Y_{\dot{\psi},m_\theta}$. $Y_{\dot{\varphi},m_\theta}$ is the FRFs between the ship roll angular velocity, $\dot{\varphi}$, and the excitation wave moment, m_θ, whereas $Y_{\dot{\psi},m_\theta}$ is the FRFs between the angular velocity of the tank free surface, $\dot{\psi}$, and the excitation wave moment, m_θ. The wave excitation moment m_θ can be represented in terms of the wave slope θ, [26], as:

$$m_\theta = a_3\theta\,, \tag{11}$$

so that the two FRFs expressed in terms of the wave slope are:

$$Y_{\dot\varphi,\theta} = a_3 Y_{\dot\varphi,m_\theta}\,, \tag{12}$$

$$Y_{\dot\psi,\theta} = a_3 Y_{\dot\psi,m_\theta}\ . \tag{13}$$

By taking **M**, **K** and **C** matrices from Eq. (4)a-c, and noting that $c_3 = b_3$, the two FRFs in Eqs. (12),(13) can be calculated as

$$Y_{\dot\varphi,\theta} = \frac{a_3 b_1 (\mathrm{i}\omega)^3 + a_3 b_2 (\mathrm{i}\omega)^2 + a_3 b_3 (\mathrm{i}\omega)}{\left(b_1 a_1 - c_1^2\right)(\mathrm{i}\omega)^4 + \left(b_2 a_1 + b_1 a_2\right)(\mathrm{i}\omega)^3 + \left(\left(a_1 - 2c_1\right)b_3 + b_2 a_2 + b_1 a_3\right)(\mathrm{i}\omega)^2 + \left(b_3 a_2 + b_2 a_3\right)(\mathrm{i}\omega) + b_3\left(a_3 - b_3\right)}\,, \tag{14}$$

$$Y_{\dot\psi,\theta} = \frac{-a_3 c_1 (\mathrm{i}\omega)^3 - a_3 b_3 (\mathrm{i}\omega)}{\left(b_1 a_1 - c_1^2\right)(\mathrm{i}\omega)^4 + \left(b_2 a_1 + b_1 a_2\right)(\mathrm{i}\omega)^3 + \left(\left(a_1 - 2c_1\right)b_3 + b_2 a_2 + b_1 a_3\right)(\mathrm{i}\omega)^2 + \left(b_3 a_2 + b_2 a_3\right)(\mathrm{i}\omega) + b_3\left(a_3 - b_3\right)}\,. \tag{15}$$

The minus sign in the tank free surface angular velocity FRF is due to use of the relative angle ψ, i.e. if the tank free surface remains parallel to the calm sea level, then the angle $\psi = -\varphi$.

2.2 The power balance

As can be seen by inspecting Eqs. (1),(2) the only dissipative terms are the total coefficient of the linear damping of the ship roll, a_2, and the coefficient of the tank linear damping, b_2. Therefore, the power input into the ship plus tank system must be a sum of the power dissipated by the ship roll and the power absorbed by the tank fluid motion:

$$P_{IN} = P_S + P_T\ . \tag{16}$$

The power dissipated by the ship roll motion can be written as:

$$P_S(\mathrm{i}\omega) = \frac{1}{2}\Re\left\{m_{a_2}^*(\mathrm{i}\omega)\,\dot\varphi(\mathrm{i}\omega)\right\}, \tag{17}$$

where $()^*$ denotes complex conjugate, and the moment m_{a_2} is the moment produced by the damping of the ship roll, which is given by:

$$m_{a_2}(\mathrm{i}\omega) = a_2\dot\varphi(\mathrm{i}\omega)\,, \tag{18}$$

so that using Eq. (17) the power dissipated by the ship roll equals:

$$P_S(\mathrm{i}\omega) = \frac{1}{2}a_2\left|\dot\varphi(\mathrm{i}\omega)\right|^2\ . \tag{19}$$

Assuming that the wave slope, θ, is represented by its spectrum, $S_{\theta,\theta}$, the mean squared value of the angular velocity of the ship roll (i.e. the expectation value of the squared velocity) can be written as:

$$E\left[\left|\dot{\varphi}\right|^2\right]=\int_{-\infty}^{\infty}\left|Y_{\dot{\varphi},\theta}\left(\mathrm{i}\omega\right)\right|^2 S_{\theta,\theta}\mathrm{d}\omega \tag{20}$$

where $E\left[\;\right]$ denotes the expectation value. The mean power dissipated by the ship roll damping is thus

$$P_S=\frac{a_2}{2}E\left[\left|\dot{\varphi}\right|^2\right]=\frac{a_2}{2}\int_{-\infty}^{\infty}\left|Y_{\dot{\varphi},\theta}\left(\mathrm{i}\omega\right)\right|^2 S_{\theta,\theta}\mathrm{d}\omega\,. \tag{21}$$

On the other hand, the power absorbed by the tank water motion is equal to that absorbed through the total coefficient of the linear damping of the tank, b_2, which can be written as:

$$P_T=\frac{1}{2}\Re\left\{m_{b_2}^{*}\left(\mathrm{i}\omega\right)\dot{\psi}\left(\mathrm{i}\omega\right)\right\}, \tag{22}$$

where the moment m_{b_2} is the moment produced by the damping of the tank fluid motion given by:

$$m_{b_2}\left(\mathrm{i}\omega\right)=b_2\dot{\psi}\left(\mathrm{i}\omega\right). \tag{23}$$

This is because $\dot{\psi}$ is the angular velocity of the tank free surface relative to the tank structure. Therefore the power absorbed by the ART equals:

$$P_T(\mathrm{i}\omega)=\frac{1}{2}b_2\left|\dot{\psi}(\mathrm{i}\omega)\right|^2. \tag{24}$$

The expectation value of the squared angular velocity of the tank fluid is:

$$E\left[\left|\dot{\psi}\right|^2\right]=\int_{-\infty}^{\infty}\left|Y_{\dot{\psi},\theta}\left(\mathrm{i}\omega\right)\right|^2 S_{\theta,\theta}\mathrm{d}\omega\,. \tag{25}$$

Thus the mean power absorber by the tank fluid motion is

$$P_T=\frac{b_2}{2}E\left[\left|\dot{\psi}\right|^2\right]=\frac{b_2}{2}\int_{-\infty}^{\infty}\left|Y_{\dot{\psi},\theta}\left(\mathrm{i}\omega\right)\right|^2 S_{\theta,\theta}\mathrm{d}\omega\,. \tag{26}$$

3 Maximisation of the power absorbed by the tank - minimisation of the roll kinetic energy

3.1 Absorbed power, dissipated power and input power

At this point it is convenient to express the steady state response of the system by using the following six dimensionless parameters:

$$f = \frac{\omega_T}{\omega_S} \qquad \eta_1 = \frac{a_2}{2\sqrt{a_1 a_3}}$$

$$\mu_1 = \frac{c_1}{\sqrt{b_1 a_1}} \qquad \eta_2 = \frac{b_2}{2\sqrt{b_1 b_3}} \tag{27)a-f \quad (}$$

$$\mu_2 = \frac{b_1}{\sqrt{b_1 a_1}} = \sqrt{\frac{b_1}{a_1}} \qquad \Omega = \frac{\omega}{\omega_S}$$

where

$$\omega_S = \sqrt{\frac{a_3}{a_1}}\,, \tag{28}$$

is the natural frequency of the ship roll assuming a “frozen” tank liquid, and

$$\omega_T = \sqrt{\frac{b_3}{b_1}}\,, \tag{29}$$

is the natural frequency of the tank fluid free oscillation in an otherwise motionless ship. The frequency ratio f is thus the ratio between the two natural frequencies, η_1 is the damping ratio of the ship roll free oscillations assuming again a frozen tank liquid, η_2 is the damping ratio of the tank fluid free oscillations assuming again a motionless ship. Furthermore, μ_1 is the frozen tank fluid rotary moment of inertia ratio, and μ_2 is the tank fluid rotary moment of inertia ratio assuming a motionless ship. Finally, Ω is the dimensionless frequency, that is, the ratio of frequency to the ship natural frequency.

Two dimensionless frequency response functions can now be defined in the form analogue to that in Eqs. (14),(15) as:

$$\Upsilon_{\dot{\varphi},\theta}(\mathrm{i}\Omega) = \omega_s Y_{\dot{\varphi},\theta}(\mathrm{i}\Omega) = \frac{B_3(\mathrm{i}\Omega)^3 + B_2(\mathrm{i}\Omega)^2 + B_1(\mathrm{i}\Omega) + B_0}{A_4(\mathrm{i}\Omega)^4 + A_3(\mathrm{i}\Omega)^3 + A_2(\mathrm{i}\Omega)^2 + A_1\mathrm{i}\Omega + A_0}\,, \tag{30}$$

$$\Upsilon_{\dot{\psi},\theta}(\mathrm{i}\Omega) = \omega_s Y_{\dot{\varphi},\theta}(\mathrm{i}\Omega) = \frac{C_3(\mathrm{i}\Omega)^3 + C_2(\mathrm{i}\Omega)^2 + C_1(\mathrm{i}\Omega) + C_0}{A_4(\mathrm{i}\Omega)^4 + A_3(\mathrm{i}\Omega)^3 + A_2(\mathrm{i}\Omega)^2 + A_1\mathrm{i}\Omega + A_0}\,, \tag{31}$$

where

$$\begin{aligned}
A_0 &= f^2 - f^4\mu_2^2 \\
A_1 &= 2f^2\eta_1 + 2\eta_2 f \\
A_2 &= 1 - (2\mu_2\mu_1 - 1)f^2 + 4\eta_2 f\eta_1 \\
A_3 &= 2\eta_2 f + 2\eta_1 \\
A_4 &= 1 - \mu_1^2
\end{aligned}
\qquad
\begin{aligned}
B_0 &= 0 \\
B_1 &= f^2 \\
B_2 &= 2\eta_2 f \\
B_3 &= 1
\end{aligned}
\qquad
\begin{aligned}
C_0 &= 0 \\
C_1 &= -f^2\mu_2 \\
C_2 &= 0 \\
C_3 &= -\mu_1
\end{aligned}
\tag{32}$$

If it is assumed that the spectrum of the wave slope, $S_{\theta,\theta}$ is independent of frequency, i.e. the wave slope spectrum has the characteristics of white Gaussian noise, the power dissipated by the ship roll expressed in terms of the dimensionless FRFs can be obtained by using (30) and (32):

$$P_S = 2\Omega_{sh}{}^4 a_1 S_{\theta,\theta}\eta_1 \int_{-\infty}^{\infty} \left|\Upsilon_{\dot{\varphi},\theta}(\mathrm{i}\Omega)\right|^2 \mathrm{d}\Omega \ . \tag{33}$$

The corresponding dimensionless power ratio, can be expressed as:

$$\Pi_S = \frac{P_S}{a_1 \omega_S{}^4 S_{\theta,\theta}} \ . \tag{34}$$

Note that the unit of $S_{\theta,\theta}$ is second, so that the denominator of (34), used to make the power index dimensionless, has the dimension of power in Watts.

Similarly, the power absorbed by the tank fluid motion expressed in terms of the dimensionless FRFs using (31) and (32) can be written as:

$$P_T = 2\, f a_1 \Omega_{sh}{}^4 \eta_2 S_{\theta,\theta} \int_{-\infty}^{\infty} \left|\Upsilon_{\dot{\psi},\theta}(\mathrm{i}\Omega)\right|^2 \mathrm{d}\Omega \ . \tag{35}$$

The corresponding dimensionless power ratio is:

$$\Pi_T = \frac{P_T}{a_1 \omega_S{}^4 S_{\theta,\theta}} \ . \tag{36}$$

The integrals in Eqs. (33) and (35) can be respectively calculated as [41]:

$$\int_{-\infty}^{\infty} \left|\Upsilon_{\dot{\varphi},\theta}(\mathrm{i}\Omega)\right|^2 \mathrm{d}\Omega = \pi \frac{A_0 B_3^2 \left(A_0 A_3 - A_1 A_2\right) + A_0 A_1 A_4 \left(2B_1 B_3 - B_2^2\right) - A_0 A_3 A_4 \left(B_1^2 - 2B_0 B_2\right) + A_4 B_0^2 \left(A_1 A_4 - A_2 A_3\right)}{A_0 A_4 \left(A_0 A_3^2 + A_1^2 A_4 - A_1 A_2 A_3\right)} , \tag{37}$$

and

$$\int_{-\infty}^{\infty} \left|\Upsilon_{\dot{\psi},\theta}(\mathrm{i}\Omega)\right|^2 \mathrm{d}\Omega = \pi \frac{\left(A_0 C_3{}^2 \left(A_0 A_3 - A_1 A_2\right) + A_0 A_1 A_4 \left(2C_1 C_3 - C_2{}^2\right) - A_0 A_3 A_4 \left(C_1{}^2 - 2C_0 C_2\right) + A_4 C_0{}^2 \left(A_1 A_4 - A_2 A_3\right)\right)}{A_0 A_4 \left(A_0 A_3{}^2 + A_1{}^2 A_4 - A_1 A_2 A_3\right)} . \tag{38}$$

By substituting from (32) into (37) and (38), and using (34) and (36) it is obtained:

$$\Pi_S = \frac{\pi \eta_1 \left(\begin{array}{l} \eta_2 \left(\mu_1{}^2 - \mu_2{}^2 - 1\right) f^4 + 4\left(\left(\mu_1{}^2 - 1\right)\eta_2{}^2 - 1/4\left(\mu_1 - \mu_2\right)^2\right)\eta_1 f^3 + \\ 4\eta_2 \left(\left(\mu_1{}^2 - 1\right)\eta_2{}^2 + 1/2\mu_2\mu_1 + 1/2 - 1/2\mu_1{}^2 - \eta_1{}^2\right) f^2 - 4\eta_1 f \eta_2{}^2 - \eta_2 \end{array} \right)}{\left(\mu_1{}^2 - 1\right) \left(\begin{array}{l} f^5 \mu_2{}^2 \eta_2{}^2 - 2\eta_1\eta_2 \left(\mu_2\mu_1 - \mu_2{}^2 - 1/2\right) f^4 + \left(\left(4\eta_1{}^2 - 2\mu_2\mu_1\right)\eta_2{}^2 + \eta_1{}^2 \left(\mu_1 - \mu_2\right)^2\right) f^3 + \\ 2\eta_1\eta_2 \left(2\eta_1{}^2 + 2\eta_2{}^2 + \mu_1{}^2 - \mu_2\mu_1 - 1\right) f^2 + \eta_2{}^2 \left(\mu_1{}^2 + 4\eta_1{}^2\right) f + \eta_1\eta_2 \end{array} \right)} \tag{39}$$

for the dimensionless index of the dissipated power for the ship, and

$$\Pi_T = -\frac{\pi f \eta_2 \left(f^4 \eta_2 \mu_2^2 + \eta_1 (\mu_1 - \mu_2)^2 f^3 + 4\mu_1 \left(\mu_1 \eta_1^2 - 1/2\mu_2 \right) \eta_2 f^2 + 4 f \eta_2^2 \mu_1^2 \eta_1 + \eta_2 \mu_1^2 \right)}{\left(\mu_1^2 - 1\right) \left(\begin{array}{l} f^5 \mu_2^2 \eta_2^2 - 2\eta_1 \eta_2 \left(\mu_2 \mu_1 - \mu_2^2 - 1/2 \right) f^4 + \left(\left(4\eta_1^2 - 2\mu_2 \mu_1 \right) \eta_2^2 + \eta_1^2 (\mu_1 - \mu_2)^2 \right) f^3 + \\ 2\eta_1 \eta_2 \left(2\eta_1^2 + 2\eta_2^2 + \mu_1^2 - \mu_2 \mu_1 - 1 \right) f^2 + \eta_2^2 \left(\mu_1^2 + 4\eta_1^2 \right) f + \eta_1 \eta_2 \end{array} \right)} \quad (40)$$

for the dimensionless absorbed power index for the tank.

The sum of the two dimensionless powers is the dimensionless power input, which using (39) and (40) greatly simplifies and becomes:

$$\Pi_{IN} = \Pi_S + \Pi_T = \frac{\pi}{1 - \mu_1^2} . \quad (41)$$

where

$$\Pi_{IN} = P_{IN} / a_1 \omega_S^4 . \quad (42)$$

It can be seen that the power input $P_{IN} = a_1 \omega_S^4 \Pi_{IN}$ does not depend on how the ART is tuned. Furthermore, the rotational inertia ratio μ_1, can be neglected in Eq. (41), since the value of c_1 is normally small in comparison with a_1 and b_1, Eq. (32), see also [26]. Therefore, it can be stated with sufficient accuracy that the power input only depends on the moment of inertia of the ship plus tanks system, a_1 multiplied by the natural frequency of the ship roll to the power of four. Since the sum $\Pi_S + \Pi_T$ is invariant, even if μ_1^2 in Eq. (41) is not neglected, it becomes clear that maximising the power absorbed by the ART, Π_T, minimises the power dissipated by the ship roll, Π_S. Given that the average kinetic energy of the roll of the ship is proportional to the mean squared angular roll velocity through the moment of inertia of the ship plus tank system, a_1, minimising the power dissipated by the ship roll is equivalent to minimising the roll kinetic energy. Therefore a maximisation of the power absorbed by the tank also leads to minimisation of the roll kinetic energy.

3.2 H_2 optimisation

A type of optimisation which minimises (or maximises) energy in signals is often referred to as H_2 optimisation. This is in contrast to the other widely used criterion, H_∞, which optimises for the maximum expected amplitudes. The combination of the frequency ratio f and the tank fluid damping ratio η_2 which maximises the power absorbed by the tank can be obtained by requiring that the derivatives of the dimensionless absorbed power index of the tank, Π_T, by the frequency ratio f and by the tank damping ratio η_2 both vanish. Alternatively, the same procedure can be used to minimise the power dissipated by the ship, Π_S which, according to Eq. (41) must yield the same result. The rotational inertia ratio μ_1, can be neglected in the expressions for the two derivatives. Also, the ship roll damping ratio, η_1 ,is normally low, especially if the ship is not equipped with bilge keels, and can also be neglected in the expressions for the two derivatives. The conditions that must be satisfied for the two simplified derivatives to vanish are:

$$\begin{aligned} \frac{\partial \Pi_T}{\partial f} = 0 \ &\Rightarrow \ 5 + \left(12\eta_2^2 - 6\right) f^2 + \left(1 + \mu_2^2\right) f^4 = 0 \\ \frac{\partial \Pi_T}{\partial \eta_2} = 0 \ &\Rightarrow \ 4 f^2 \eta_2^2 + 2 f^2 - 1 - \left(1 + \mu_2^2\right) f^4 = 0. \end{aligned} \quad (43)$$

Solving (43) simultaneously for f and η_2 results in eight roots, of which the physically meaningful one for a typical ship plus ART system, which maximises the power absorbed by the ART is:

$$
\begin{aligned}
f_{OPT} &= \frac{\sqrt{2\left(1+\mu_2{}^2\right)\left(3-\sqrt{1-8\mu_2{}^2}\right)}}{2(1+\mu_2{}^2)}, \\
\eta_{2,OPT} &= \frac{\sqrt{1-\sqrt{1-8\mu_2{}^2}}}{4}.
\end{aligned}
\tag{44}
$$

The validity of the expressions (44) is limited to the inertia ratios μ_2 below the ratios of at most 35%. However the inertia ratio μ_2 is normally less than that, since the moment of inertia of the tank fluid, b_1, is smaller than the moment of inertia of the ship plus frozen tank, a_1. In fact, given that μ_2^2 is small in comparison to one, the optimum frequency ratio is just above unity. It is a physically sound and acceptable result that the best power absorption by the tank is obtained by tuning its natural frequency close to the natural frequency of the ship roll, $f_{OPT} \approx 1$. Even when applying different optimisation criteria on other types of oscillation absorber systems [34,36–40] the optimum natural frequency ratio close to unity has been obtained.

An illustrative example of a ship equipped with a U-tube ART is considered next. The system is characterised by the following dimensionless parameters: $\mu_1 = 0.048$, $\mu_2 = 0.192$, and $\eta_1 = 0.075$. The example ship is taken from [26]. It is assumed for convenience that $S_{\theta,\theta} = 1$ s, and $\omega_S = 1\ \mathrm{s}^{-1}$. Two tuning criteria of the ART are compared, the H_∞ criterion, aiming at minimising the maximum roll angle, and the present H_2 criterion, aiming at maximising the power absorbed by the ART. It is seen in Figure 2 that both tuning criteria significantly reduce either the roll angle or the roll angular velocity squared in comparison to the situation where the ship is not equipped with an ART.

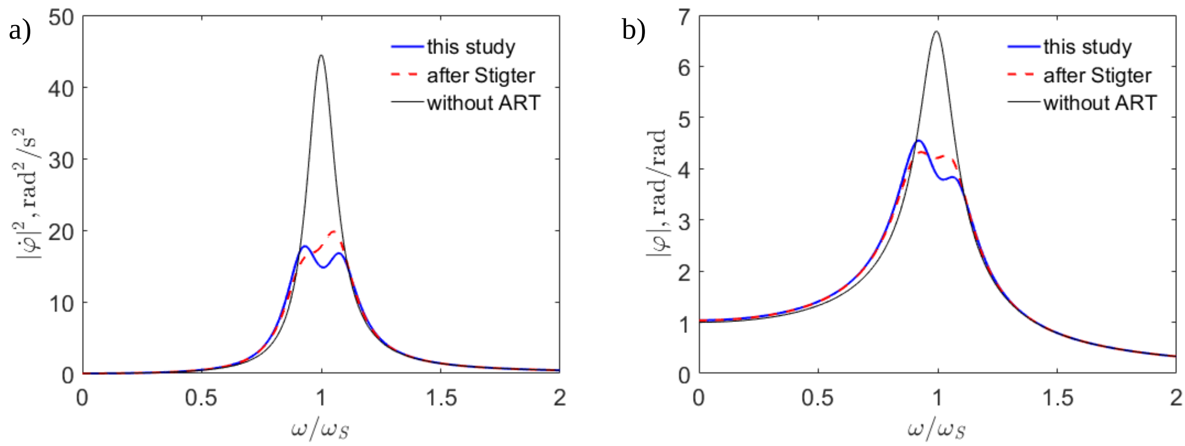

Figure 2: a) the roll velocity squared b) the roll angle as a function of frequency per unit excitation wave angle

The area under the solid curve in Figure 2a) is in fact minimised by using the optimum tuning parameters of Eq. (44). If interpreted in terms of the maximum ship roll angle, Figure 2 b), it is seen that the present tuning is somewhat inferior to the tuning using the H_∞ criterion, as one would expect, given the different aims of the two tuning principles. Still, the increased roll angle near the first natural frequency of the H_2 optimised ship plus ART system is not excessive, as seen by comparing the solid line with the dashed line at the frequency of about $0.9\omega_S$.

Figure 3a) shows the dimensionless index of the power dissipated by the ship and Figure 3b) shows the dimensionless index of the power absorbed by the ART.

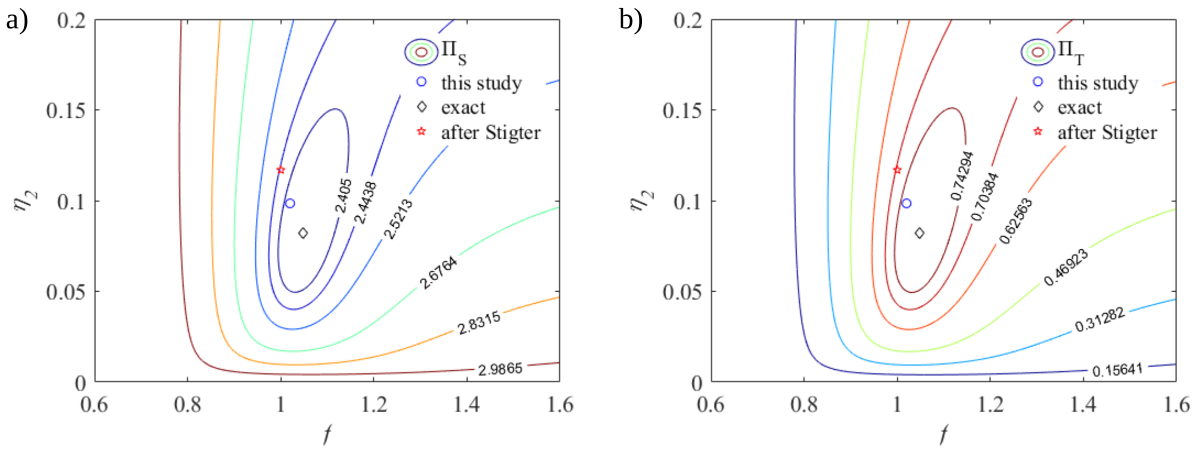

Figure 3: a) The dimensionless absorbed power index of the tank and b) the dimensionless index of the power dissipated by the ship, as functions of the frequency ratio and the ART damping ratio with $\mu_1 = 0.046$, and $\eta_1 = 0.075$

Both powers are calculated using Eqs. (39),(40) and plotted are as functions of the frequency ratio and the ART damping ratio. It can be seen that the maximum of the power absorbed indeed corresponds to the minimum of the power dissipated, as shown by the diamond markers in the two plots. Also note that the levels of corresponding contours in each plot sum to approximately π. However, the optimum damping-frequency ratio pair, calculated using Eqs. (44) and designated by the circle, does not exactly correspond to the extreme values calculated by numerically maximising Π_T (minimising Π_S) of the two power indices, designated by the diamond marker.

This is due to the combined effects of neglecting the ship roll damping ratio, η_1, and the moment of inertia ratio, μ_1. In order to illustrate that the discrepancy is due to the neglects made, a situation is considered in which the ship roll damping ratio, η_1, and the moment of inertia ratio, μ_1, are both reduced by a factor of ten.

As shown by Figure 4, the optimum damping-frequency ratio pair, calculated using Eqs. (44) now overlaps with the exact extreme values of the two power indices. The optimum tuning pair obtained using the H_∞ criterion on the roll angle is designated by the pentagon markers in Figure 3, indicating somewhat smaller optimum damping and frequency ratios than those obtained in this study. Both criteria suggest the frequency ratio of about unity and a tank damping ratio of about 10% for the example ship considered.

The absorbed power is in fact mostly dissipated through the boundary layer friction between the tank fluid and its walls, the vortices that occur due to the non-ideal flow of the water in the two reservoirs, and the losses in the “knee” between the reservoirs and the connection conduit. Therefore only a fraction of total absorbed power could be converted to, for example, electricity. This could be done by integrating a turbine in the channel connecting the port and starboard reservoirs of the ART that drives an electrical generator. If the generator is shunted with a variable electrical load, then, even if this load is a pure resistance, the temperature developed on the load could be monitored in order to maximise the power absorbed, by varying the load. This could potentially enable an automatic tuning of the damping ratio of the ART which would ensure a maximum power absorbed as the conditions at sea change. As discussed in Section 2 this means also a minimum of the expected value of the roll angular velocity which is a valid criterion for the performance of roll control techniques.

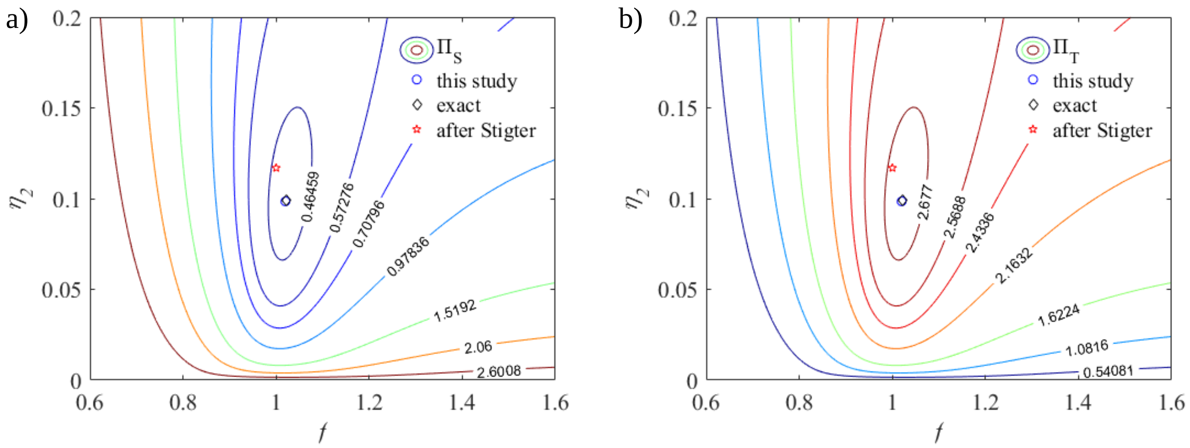

Figure 4: a) The dimensionless absorbed power index of the tank and b) the dimensionless index of the power dissipated by the ship, as functions of the frequency ratio and the ART damping ratio with $\mu_1 = 0.0046$, and $\eta_1 = 0.0075$

4 Conclusions

In this paper it is investigated how the natural frequency and the damping ratio of an ART should be tuned to maximise the power absorbed by the tank. It is also studied how this tuning corresponds to a tuning that minimises the kinetic energy of the ship roll. It is found that the two tuning criteria are exactly equivalent assuming the flat spectrum of the roll moment. This is a result of the fact that the power input of the wave excitation moment into the ship plus tank system does not depend on how the ART is tuned. Rather, different tunings of the ART natural frequency and damping ratio only affect the distribution of the power dissipated by the ship roll damping and the power absorbed by the ART. The optimum tuning parameters, the natural frequency and the damping ratio of the ART are calculated analytically assuming the flat spectrum of the wave excitation moment. The optimum tuning parameters are found not to be particularly different from those minimising the maximum roll angle for the example ship considered. The power input into the ship plus tank system is found to be proportional to the natural frequency of the ship roll to the power of four multiplied by the moment of inertia of the ship for the flat spectrum of the wave slope. According to this theoretical result, if specialised offshore installations are considered for harvesting the wave energy by using U-tube ARTs, they should be designed with an as high roll natural frequency as possible. Finally, it is important to emphasize that the study is carried out using a linearized model in which a number of effects has been neglected, like, for example, sloshing of the fluid in the ART.

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